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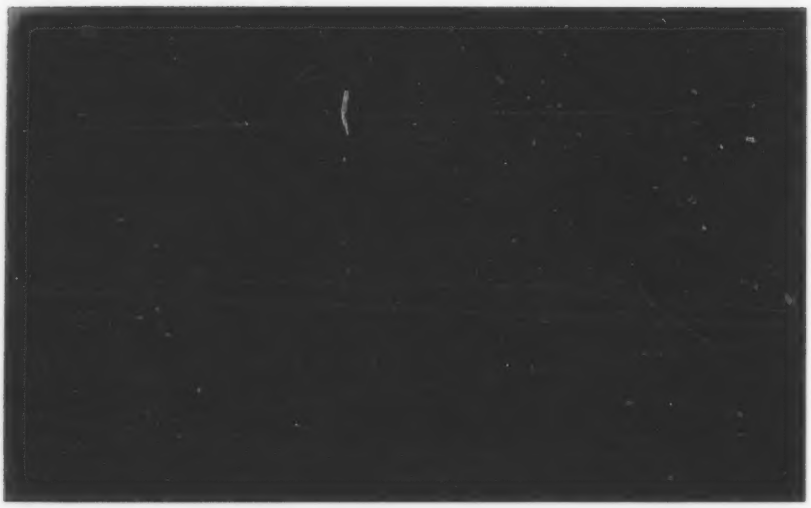

CENTRE DE RECHERCHE SUR LES TRANSPORTS
UNIVERSITÉ DE MONTRÉAL
C.P. 6128, SUCCURSALE A
MONTRÉAL
CANADA H3C 3J7

TÉLÉPHONE : (514) 343-7575
EMAIL : crt@crt.umontreal.ca
TÉLÉCOPIEUR (FAX) : (514) 343-7121

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Dual-Ascent Procedures for Multicommodity Location/Allocation Problems with Balancing Requirements

by

Teodor Gabriel Crainic[1]
Louis Delorme[2]

[1] Département des sciences administratives, Université du Québec à Montréal, and Centre de recherche sur les transports, Université de Montréal, C.P. 6128, succursale A, Montréal, Québec H3C 3J7

[2] I.R.E.Q., Hydro-Québec, Méthodes numériques, 1800, montée Sainte-Julie, Varennes, Québec J3X 1S1

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ABSTRACT

The multicommodity location-allocation problem with balancing requirements is related to one of the major logistics issues faced by distribution and transportation firms: the management of the fleet of vehicles over a medium to long term planning horizon. We present a dual-ascent approach for this formulation which may be used either to obtain a good heuristic solution, or to compute sharp bounds in a branch-and-bound scheme. We present and compare two variants of this approach, and analyze their performances both on a large number of randomly generated problems and on a large-scale application to the strategic/tactical planning of the land operations of an international maritime container shipping firm. Our results demonstrate that a dual-ascent approach is highly effective in this context.

KEY WORDS: DUAL-ASCENT ALGORITHMS,
MULTICOMMODITY LOCATION-ALLOCATION MODELS,
EMPTY FLOW BALANCING, FLEET MANAGEMENT,
CONTAINER TRANSPORTATION.

RESUME

Le problème de localisation-allocation avec échanges inter-dépôts (ou exigences d'équilibrage) est associé à un des aspects importants de la planification logistique des entreprises de transport et de distribution, à savoir la gestion de la flotte de véhicules à moyen et long termes. Nous présentons une approche algorithmique de type "dual ascendant" qui peut être utilisée soit per se pour obtenir une bonne solution approchée, soit pour calculer des bornes efficaces dans un algorithme d'énumération de type "branch-and-bound". Nous présentons et comparons deux variantes de cette approche, et nous analysons leur comportement à partir d'un ensemble de problèmes générés aléatoirement et d'une application de grande taille au problème de planification stratégique/tactique des opérations terrestres d'une compagnie maritime internationale de transport de conteneurs. Les résultats obtenus montrent que l'approche duale ascendante est extrêmement efficace dans ce contexte.

MOTS CLES: ALGORITHMES DUAUX-ASCENDANTS,
MODELES DE LOCALISATION-ALLOCATION MULTIPRODUITS,
GESTION DE FLOTTE, MOUVEMENTS A VIDE,
TRANSPORT DES CONTENEURS.

INTRODUCTION

The management of the fleet of vehicles over a medium to long term planning horizon constitutes one of the major logistics issues faced by distribution and transportation firms. This strategic/tactical planning problem is particularly concerned with the location of the depots for the rolling stock and the forecast and management of the empty movements.

Maritime container transportation companies face a particularly challenging version of this problem when contemplating the planning of their land operations. This problem may be stated as follows: Once a ship arrives at port, the shipping company has to deliver loaded containers to their designated in-land destinations. These containers come in several types and sizes, and various transportation modes may be used to move them. Following their delivery and unloading by the importing customer, empty containers may be returned to the depot of the port that initiated the loaded movement. However, they may also be repositioned to a different depot in anticipation of future requests for empty containers. Indeed, the same, or other, customers are also requesting empty containers of specific types for subsequent shipping of their own products. Empty containers are delivered from depots, are loaded, and are subsequently moved, by land and maritime transportation means, to their final destinations. For a full description and discussion of this problem, see Dejax, Crainic and Delorme, 1992.

A major complication of the problem results from the regional imbalances in empty container availabilities and needs throughout the network: some areas lack containers of certain types, while others have surpluses of them. This requires that empty containers be shipped between depots to "balance" the network. Note that interdepot transportation is assumed to be more efficient per moved unit than movements between depots and customers, due to the larger volumes of traffic which may be consolidated (e.g. unit trains) for transshipment over relatively long distances. This also justifies the very existence of interdepot movements (otherwise, empty containers would be positioned directly to the requesting depots). This differentiates this problem from a classical location-allocation application.

The general problem is therefore to locate the empty container depots in order to collect the supply of empty containers available at customers' sites and to satisfy the customer requests for empty containers, while minimizing the total operating cost: the cost of opening and operating the depots, plus the transportation costs between customers and depots and the

costs generated by moving empty containers among depots to "balance" the network. It may be interesting to note that, in practice, this strategic/tactical logistics problem may have to be solved repeatedly since container shipping companies do not often build their own depots; rather, they use facilities available from other modes (e.g. ports and rail yards). It is then a question of deciding which facilities to use, according to the demand and the cost structure particular to the specific planning period. It is noteworthy that this last characteristic represents another difference with the usual location-allocation application. It also explains why, in practice, fixed depot costs may be rather low.

We have already proposed (Crainic, Dejax and Delorme, 1989) a modeling framework for this class of problems, that we call the *simple multicommodity location-allocation problem with balancing requirements (SLBP)*. Here, *simple* does not imply a "single echelon" formulation since the problem may actually be described as being "two echelon." Rather, it refers to the usual hypotheses concerning this problem, which assume that depot capacity does not restrict depot utilization, that transportation and depot operating costs are linear, and that supplies and demands are deterministic. (The first assumption may be easily verified in actual practice, while the second reflects the data and costing definitions which are currently available in industry.) The presence of balancing requirements represents one of the main characteristics of the problem, since it unveils a network flow structure that is most favorable to efficient algorithmic developments. It also constitutes a major difference between this formulation and classical location models, as reflected in the behaviour of the branch-and-bound algorithm developed by Crainic, Delorme and Dejax, 1991.

In this paper we present a dual-ascent based approach for solving simple multicommodity location-allocation problems with balancing requirements. In the first section, we restate the general formulation and some important properties, and briefly mention previous algorithmic approaches proposed for this problem. Section 2 is dedicated to the presentation of the dual-ascent approach. First the dual formulation of the strong linear relaxation of the SLBP is stated. Two subproblems are then derived by fixing suitably chosen dual variables. Each of these subproblems allows us to compute lower and upper bounds on the optimal solution of the SLBP, as well as to derive a feasible solution for the multicommodity location-allocation problem with balancing requirements. Based on these results, two dual-ascent procedures are presented which may be used either to obtain a heuristic solution of the SLBP, or

to compute sharp bounds in a branch-and-bound scheme. We tested these procedures both on a wide gamut of randomly generated problems and on a large scale application. Section 3 presents the results of this experimentation.

In the past, dual-ascent methods have proved successful for several difficult problems; see Erlenkotter (1978), Van Roy and Erlenkotter (1982), Klincewicz and Luss (1987), Wong (1984), and Balakrishnan, Magnanti and Wong (1989), for example. Our results prove that this approach is equally effective for large scale, multicommodity location-allocation problems with balancing requirements.

1. THE MODEL

Let C, D and P be the sets of customers (cardinality m), candidate depots (cardinality n) and products (cardinality l), respectively. Let

f_j : "Fixed" cost of depot j, $j \in D$;

c_{ijp}, c_{jip} : Unit transportation costs (in both directions) for product p, $p \in P$, between customer i, $i \in C$, and depot j, $j \in D$;

s_{jkp}, s_{kjp} : Unit transportation costs for product p, $p \in P$, between depots j and k, $j \in D$, $k \in D$;

O_{ip} : Supply at node (customer) i, $i \in C$, of product p, $p \in P$;

D_{ip} : Demand at node (customer) i, $i \in C$, of product p, $p \in P$.

Define the following decision variables:

$y = (y_j)$: Opening vector for depots, $j \in D$; y_j equals 1 if depot j is open and zero otherwise;

x_{ijp}, x_{jip} : Flows (in both directions) of product p, $p \in P$, between customer i, $i \in C$, and depot j, $j \in D$;

w_{jkp}, w_{kjp} : Flows of product p, $p \in P$, between depots j and k, $j \in D$, $k \in D$.

Crainic, Dejax and Delorme (1989) then formulate the model as:

$$\textit{Minimize} \quad Z = \sum_{j \in D} f_j \, y_j + \sum_{p \in P} \left\{ \sum_{i \in C} \sum_{j \in D} (c_{ijp} \, x_{ijp} + c_{jip} \, x_{jip}) + \sum_{j \in D} \sum_{k \in D} s_{jkp} \, w_{jkp} \right\}$$

$$\textit{Subject to} \quad \sum_{j \in D} x_{ijp} = O_{ip} \quad \text{and} \quad \sum_{j \in D} x_{jip} = D_{ip} \qquad \forall \, i \in C, p \in P \tag{1}$$

$$x_{ijp} \le M \, y_j \quad \text{and} \quad x_{jip} \le M \, y_j \qquad \forall \, i \in C, j \in D, p \in P \tag{2}$$

$$w_{jkp} \le M \, y_j \quad \text{and} \quad w_{kjp} \le M \, y_j \qquad \forall \, j \in D, k \in D, p \in P \tag{3}$$

$$\sum_{i \in C} x_{ijp} + \sum_{k \in D} w_{kjp} - \sum_{i \in C} x_{jip} - \sum_{k \in D} w_{jkp} = 0 \qquad \forall \, j \in D, p \in P \tag{4}$$

$$x_{ijp} \ge 0 \quad \text{and} \quad x_{jip} \ge 0 \qquad \forall \, i \in C, j \in D, p \in P \tag{5}$$

$$w_{jkp} \ge 0 \qquad \forall \, j \in D, k \in D, p \in P \tag{6}$$

$$y_j \in \{0,1\} \qquad \forall\, j \in D. \tag{7}$$

Constraints (1) ensure that demand and supply requirements are met. Constraints (2) forbid exchanges between customers and closed depots (M is a suitably large positive constant), while constraints (3) have a similar role for interdepot movements. Constraints (4) correspond to the interdepot balancing requirements. Figure 1 illustrates the structure of the problem.

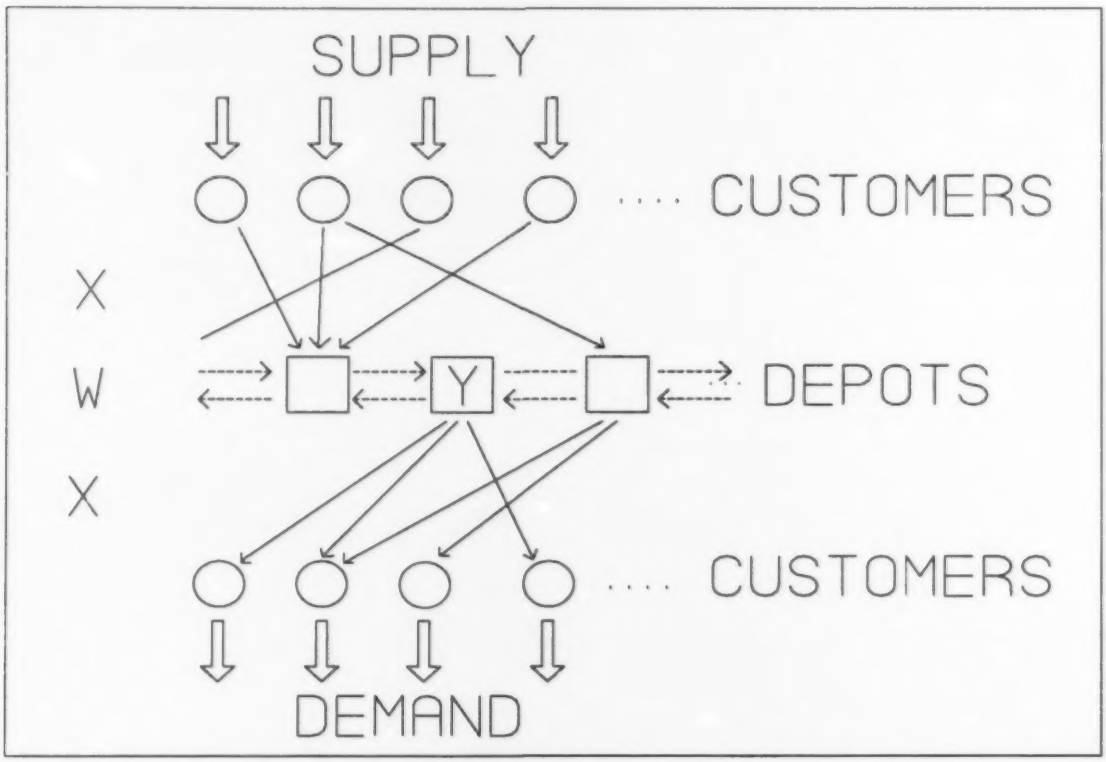

Figure 1. Structure of the SLBP

As mentioned in the introduction, we assume that for the range of distances characterizing them, interdepot movements are more efficient, per transported unit, than movements between customers and depots. This forms the rationale for the problem and, by insuring that interdepot movements do take place, contributes towards the interest and originality of the model. Indeed, Crainic, Dejax and Delorme (1989) have shown that when this hypothesis is not verified, in particular when

$$c_{ijp} + s_{jkp} \geq c_{ikp} \qquad \forall\, i \in C, j \in D, k \in D, p \in P,$$

there exists an optimal solution with $w_{jkp} = 0$ for all $j \in D, k \in D, p \in P$. On the other hand,

they have also shown that, when interdepot transportation costs satisfy the triangle inequality

$$s_{jkp} \leq s_{jrp} + s_{rkp} \quad \forall\, j \in D, k \in D, r \in D,$$

constraints (3) are redundant, and the model is more concisely written as:

$$(L)\ \textit{Minimize}\ \ Z = \sum_{j \in D} f_j\, y_j + \sum_{p \in P} \left\{ \sum_{i \in C} \sum_{j \in D} (C_{ijp}\, x_{ijp} + C_{jip}\, x_{jip}) + \sum_{j \in D} \sum_{k \in D} s_{jkp}\, w_{jkp} \right\}$$

$$\textit{Subject to} \quad \sum_{j \in D} x_{ijp} = 1 \qquad \forall\, i \in C, p \in P$$

$$\sum_{j \in D} x_{jip} = 1 \qquad \forall\, i \in C, p \in P \qquad \text{(L1)}$$

$$x_{ijp} \leq y_j \qquad \forall\, i \in C, j \in D, p \in P$$

$$x_{jip} \leq y_j \qquad \forall\, i \in C, j \in D, p \in P \qquad \text{(L2)}$$

$$\sum_{i \in C} O_{ip}\, x_{ijp} + \sum_{k \in D} w_{kjp} - \sum_{i \in C} D_{ip}\, x_{jip} - \sum_{k \in D} w_{jkp} = 0 \quad \forall\, j \in D, p \in P \qquad \text{(L3)}$$

$$x_{ijp}, x_{jip} \geq 0 \qquad \forall\, i \in C, j \in D, p \in P \qquad \text{(L4)}$$

$$w_{jkp} \geq 0 \qquad \forall\, j \in D, k \in D, p \in P \qquad \text{(L5)}$$

$$y_j \in \{0,1\} \qquad \forall\, j \in D \qquad \text{(L6)}$$

where $C_{ijp} = O_{ip} c_{ijp}$ and $C_{jip} = D_{ip} c_{jip}$, $\forall\ i \in C, j \in D, p \in P$. Since this hypothesis is rather mild and is easily verified in practice, it is this last formulation which is usually solved. Note that we have stated the *normalized* version of model, which is used throughout the rest of this paper.

A few algorithmic developments have already been spurred by this problem. The exploration of classical heuristic approaches (greedy, interchange, etc.) has been initiated with encouraging success (Guo, 1990). Lagrangian relaxation approaches have proven quite efficient for moderate sized problems, but their performance seems to deteriorate with the number of commodities (Guo, 1990; Guo et al., 1989). Research is continuing in this direction, as well as in the area of primal/dual approaches (Tétreault, 1990). Crainic, Delorme and Dejax (1991) propose a branch-and-bound algorithm based on the weak relaxation of the problem and show, in particular, that the classical results for the simple plant location problem (Krarup and Pruzan, 1983) do not extend to the SLBP. Indeed, branching rules which are efficient in the classical case (e.g., Khumawala's largest-Ω rule) are among the worst for the

present problem. The authors, as well as Gendron and Crainic (1992), then proceed to develop and analyze specific bounding and branching rules aimed at obtaining an efficient branch-and-bound framework for the SLBP. We use their results to analyze the precision of the dual-ascent procedure. An interesting new development is the work of Crainic, Gendreau, Soriano and Toulouse (1991) on an algorithm based on Tabu Search principles (Glover, 1989, 1990) for the mixed-integer formulation of the SLBP. The authors obtain results which compare advantageously to those obtained by using the dual-ascent procedures presented in this paper. This shows that Tabu Search may successfully be applied to solve mixed-integer problems with a significant "continuous" component. Moreover, it indicates that Tabu Search may offer an interesting approach for problems for which good heuristics are less readily available (Farvolden, Gendreau and Crainic, 1991).

2. DUAL-ASCENT ALGORITHMS

Let us consider the *strong linear relaxation* of problem (L):

(P) *Minimize* $Z(y, x, w) =$

$$\sum_{j \in D} f_j \, y_j + \sum_{p \in P} \left\{ \sum_{i \in C} \sum_{j \in D} (C_{ijp} \, x_{ijp} + C_{jip} \, x_{jip}) + \sum_{j \in D} \sum_{k \in D} s_{jkp} \, w_{jkp} \right\}$$

Subject to

$$\sum_{j \in D} x_{ijp} = 1 \qquad \forall \, i \in C, p \in P \qquad (u_{ip})$$

$$\sum_{j \in D} x_{jip} = 1 \qquad \forall \, i \in C, p \in P \qquad (v_{ip}) \qquad \text{(P1)}$$

$$x_{ijp} \leq y_j \qquad \forall \, i \in C, j \in D, p \in P \qquad (\gamma_{ijp})$$

$$x_{jip} \leq y_j \qquad \forall \, i \in C, j \in D, p \in P \qquad (\gamma_{jip}) \qquad \text{(P2)}$$

$$\sum_{i \in C} O_{ip} \, x_{ijp} + \sum_{k \in D} w_{kjp} - \sum_{i \in C} D_{ip} \, x_{jip} - \sum_{k \in D} w_{jkp} = 0 \quad \forall \, j \in D, p \in P \quad (\lambda_{jp}) \qquad \text{(P3)}$$

$$x_{ijp}, x_{jip} \geq 0 \qquad \forall \, i \in C, j \in D, p \in P \qquad \text{(P4)}$$

$$w_{jkp} \geq 0 \qquad \forall \, j, k \in D, p \in P \qquad \text{(P5)}$$

$$y_j \geq 0 \qquad \forall \, j \in D \qquad \text{(P6)}$$

The dual formulation of the above problem may be stated as follows:

(D) *Maximize* $Z(u, v, \gamma, \lambda) = \sum_{p \in P} \sum_{i \in C} (u_{ip} + v_{ip})$

Subject to

$$u_{ip} - \gamma_{ijp} + O_{ip} \, \lambda_{jp} \leq C_{ijp} \qquad \forall \, i \in C, j \in D, p \in P$$

$$v_{ip} - \gamma_{jip} - D_{ip} \, \lambda_{jp} \leq C_{jip} \qquad \forall \, i \in C, j \in D, p \in P \qquad \text{(D1)}$$

$$\sum_{p \in P} \sum_{i \in C} (\gamma_{ijp} + \gamma_{jip}) \leq f_j \qquad \forall \, j \in D \qquad \text{(D2)}$$

$$\lambda_{jp} - \lambda_{kp} \leq s_{kjp} \qquad \forall \, j, k \in D, p \in P \qquad \text{(D3)}$$

$$\gamma_{ijp}, \gamma_{jip} \geq 0 \qquad \forall \, i \in C, j \in D, p \in P \qquad \text{(D4)}$$

In this formulation, u_{ip} and v_{ip} are the dual variables associated to the supply and demand constraints, respectively, for each customer and for each commodity. Dual variables γ_{ijp} and γ_{jip} correspond to the "location feasibility" constraints (P2) for each direction of operation between a depot and a customer, while the λ_{jp} variables are associated to the balancing constraints for each commodity at each depot.

2.1 Dual Properties and Subproblems

To solve our problem, we intend to follow a strategy which consists of iteratively modifying the values of the dual variables in order to increase the value of the dual objective $Z(u,v,\gamma,\lambda)$. This allows us to efficiently compute lower and upper bounds on Z^*, the value of the objective function of the original problem, bounds which may then be used either to guide an enumeration algorithm, or to qualify heuristic solutions.

Observe that, when the λ_{jp} variables are set to values $\bar{\lambda}_{jp}$ which satisfy

$$\bar{\lambda}_{jp} - \bar{\lambda}_{kp} \leq s_{kjp} \qquad \forall\, j,k \in D, p \in P,$$

the following formulation is obtained:

$(D_1(\bar{\lambda}))$ *Maximize* $Z(u,v,\gamma \mid \bar{\lambda}) = \sum_{p\in P}\sum_{i\in C}(u_{ip} + v_{ip})$

Subject to

$$u_{ip} - \gamma_{ijp} \leq (C_{ijp} - O_{ip}\,\bar{\lambda}_{jp}) \qquad \forall\, i \in C, j \in D, p \in P$$

$$v_{ip} - \gamma_{jip} \leq (C_{jip} + D_{ip}\,\bar{\lambda}_{jp}) \qquad \forall\, i \in C, j \in D, p \in P$$

$$\sum_{p\in P}\sum_{i\in C}(\gamma_{ijp} + \gamma_{jip}) \leq f_j \qquad \forall\, j \in D$$

$$\gamma_{ijp}, \gamma_{jip} \geq 0 \qquad \forall\, i \in C, j \in D, p \in P$$

The dual of this formulation is

$(P_1(\bar{\lambda}))$ *Minimize* $Z(y,x \mid \bar{\lambda}) =$

$$\sum_{j\in D} f_j\, y_j + \sum_{p\in P}\sum_{i\in C}\sum_{j\in D}\left\{(C_{ijp} - O_{ip}\,\bar{\lambda}_{jp})\,x_{ijp} + (C_{jip} + D_{ip}\,\bar{\lambda}_{jp})\,x_{jip}\right\}$$

Subject to

$$\sum_{j\in D} x_{ijp} = 1 \quad \text{and} \quad \sum_{j\in D} x_{jip} = 1 \qquad \forall\, i\in C, p\in P$$

$$x_{ijp} \le y_j \quad \text{and} \quad x_{jip} \le y_j \qquad \forall\, i\in C, j\in D, p\in P$$

$$x_{ijp} \ge 0 \quad \text{and} \quad x_{jip} \ge 0 \qquad \forall\, i\in C, j\in D, p\in P$$

$$y_j \ge 0 \qquad \forall\, j\in D$$

This formulation clearly is the strong linear relaxation of a simple plant location problem (Krarup and Pruzan, 1983), that we note $(L_1(\bar{\lambda}))$. Here, the multicommodity aspect of the problem, as well as the fact that customers may both send to and receive items from depots, does not increase the difficulty of the problem. It does, however, affect the implementation of the solution procedures and it increases the data storage requirements.

The $\bar{\lambda}$ variables characterize the state of "imbalance" of the system at each depot. Indeed, if $\bar{\lambda}_{jp}$ is increased then the transportation costs associated with x_{ijp} in problem $P_1(\bar{\lambda})$ decrease, while those of variables x_{jip} increase. Hence, when $\bar{\lambda}_{jp}$ is relatively large and positive, depot j attracts more flow than it sends to customers. The same is true, mutatis mutandis, for large, negative values of $\bar{\lambda}_{jp}$. Therefore, large values for some $\bar{\lambda}$ tend to indicate that the corresponding depots have important shortages or surpluses of certain commodities, which result in interdepot balancing flows.

Problems $(D_1(\bar{\lambda}))$, $(P_1(\bar{\lambda}))$, $(L_1(\bar{\lambda}))$ and their solutions may be used to determine a feasible solution to problem (L) and to compute lower and upper bounds on Z^*. Let $(\bar{u}, \bar{v}, \bar{\gamma})$ be the optimal solution to problem $(D_1(\bar{\lambda}))$, with $Z(\bar{u}, \bar{v}, \bar{\gamma} \mid \bar{\lambda})$ the associated objective function value, and $(\bar{y}, \bar{x})$ the corresponding feasible (integer) solution to problem $(L_1(\bar{\lambda}))$. (See Van Roy and Erlenkotter, 1982, for one possible way to construct such a feasible solution.) Consider

$$(L(\bar{y}, \bar{x}, w)) \quad \textit{Minimize} \quad Z(w \mid \bar{y}, \bar{x}) = \sum_{p\in P}\sum_{j\in D}\sum_{k\in D} s_{jkp}\, w_{jkp}$$

$$\textit{Subject to} \quad \sum_{k\in D} w_{kjp} - \sum_{k\in D} w_{jkp} = \sum_{i\in C} D_{ip}\, \bar{x}_{jip} - \sum_{i\in C} O_{ip}\, \bar{x}_{ijp} \qquad \forall\, j\in D, p\in P$$

$$w_{jkp} \ge 0 \qquad \forall\, j,k\in D, p\in P$$

which is the original problem with variables y and x set to values $\bar{y}$ and $\bar{x}$, respectively. Problem $(L(\bar{y}, \bar{x}, w))$ is a multicommodity transportation problem, which decomposes into l mutually exclusive minimum cost transportation problems, one for each commodity. The origin and destination nodes of this problem correspond to the open depots in the solution to problem $(L_1(\bar{\lambda}))$, while the corresponding supplies and demands are computed according to the allocation of customers to depots defined by this same solution. Let $\hat{w}$ be its optimal solution and $Z(\bar{y}, \bar{x}, \hat{w})$ the corresponding objective function value. Then

Proposition 1. $Z(\bar{u}, \bar{v}, \bar{\gamma} \mid \bar{\lambda}) \leq Z^* \leq Z(\bar{y}, \bar{x}, \hat{w})$.

To justify this result, note that $(\bar{y}, \bar{x}, \hat{w})$ forms a feasible solution to problem (L); hence, $Z^* \leq Z(\bar{y}, \bar{x}, \hat{w})$. The other side of the proposition follows directly from linear duality arguments.

A possibly tighter bound may however be derived. Note that a feasible solution to problem (L) may also be obtained by setting the y variables to $\bar{y}$ and then by solving

$(L(\bar{y}, x, w))$ *Minimize* $Z(w, x \mid \bar{y}) =$

$$\sum_{p \in P} \left\{ \sum_{i \in C} \sum_{j \in D} (C_{ijp}\, x_{ijp} + C_{jip}\, x_{jip}) + \sum_{j \in D} \sum_{k \in D} s_{jkp}\, w_{jkp} \right\}$$

Subject to

$$\sum_{j \in D} x_{ijp} = 1 \qquad \forall\, i \in C, p \in P$$

$$\sum_{j \in D} x_{jip} = 1 \qquad \forall\, i \in C, p \in P$$

$$\sum_{i \in C} O_{ip}\, x_{ijp} + \sum_{k \in D} w_{kjp} - \sum_{i \in C} D_{ip}\, x_{jip} - \sum_{k \in D} w_{jkp} = 0 \quad \forall\, j \in D, p \in P$$

$$\bar{y}_j \geq x_{ijp}, x_{jip} \geq 0 \qquad \forall\, i \in C, j \in D, p \in P$$

$$w_{jkp} \geq 0 \qquad \forall\, j, k \in D, p \in P$$

Problem ($L(\bar{y}, x, w)$) is a multicommodity minimum cost network flow problem, which also decomposes into l mutually exclusive minimum cost transshipment problems. Furthermore, it is a relaxation of problem ($L(\bar{y}, \bar{x}, w)$), in the sense that it considers the same open depots but it allows the reassignment of customers to depots. Let $(\bar{\bar{x}}, \bar{\bar{w}})$ be the optimal solution of ($L(\bar{y}, x, w)$) and $Z(\bar{y}, \bar{\bar{x}}, \bar{\bar{w}})$ the corresponding objective function value. We then have

Proposition 2. $Z(\bar{y}, \bar{x} \mid \bar{\lambda}) \leq Z^* \leq Z(\bar{y}, \bar{\bar{x}}, \bar{\bar{w}}) \leq Z(\bar{y}, \bar{x}, \hat{w})$.

Note that similar bounds, although less tight, may be derived from any feasible solution (u, v, γ) to problem ($D_1(\bar{\lambda})$). We use this fact in the solution procedures presented at the end of the section.

A second set of bounds on Z^* may be derived from (D). Observe that, when variables γ_{ijp} and γ_{jip} are fixed to values $\tilde{\gamma}_{ijp}$ and $\tilde{\gamma}_{jip}$ which satisfy

$$\sum_{p \in P} \sum_{i \in C} (\tilde{\gamma}_{ijp} + \tilde{\gamma}_{jip}) \leq f_j \qquad \forall\, j \in D$$

$$\tilde{\gamma}_{ijp}, \tilde{\gamma}_{jip} \geq 0 \qquad \forall\, i \in C, j \in D, p \in P,$$

the original dual problem (D) reduces to

($D_2(\tilde{\gamma})$) *Maximize* $Z(u, v, \lambda \mid \tilde{\gamma}) = \sum_{p \in P} \sum_{i \in C} (u_{ip} + v_{ip})$

Subject to

$$u_{ip} + O_{ip}\, \lambda_{jp} \leq C_{ijp} + \tilde{\gamma}_{ijp} \qquad \forall\, i \in C, j \in D, p \in P$$

$$v_{ip} - D_{ip}\, \lambda_{jp} \leq C_{jip} + \tilde{\gamma}_{jip} \qquad \forall\, i \in C, j \in D, p \in P$$

$$\lambda_{jp} - \lambda_{kp} \leq s_{kjp} \qquad \forall\, j, k \in D, p \in P$$

The dual of the above problem is

($P_2(\tilde{\gamma})$) *Minimize* $Z(x, w \mid \tilde{\gamma}) =$

$$\sum_{p \in P} \left\{ \sum_{i \in C} \sum_{j \in D} ((C_{ijp} + \tilde{\gamma}_{ijp})\, x_{ijp} + (C_{jip} + \tilde{\gamma}_{jip})\, x_{jip}) + \sum_{j \in D} \sum_{k \in D} s_{jkp}\, w_{jkp} \right\}$$

Subject to

$$\sum_{j \in D} x_{ijp} = 1 \qquad \forall\, i \in C, p \in P$$

$$\sum_{j \in D} x_{jip} = 1 \qquad \forall\, i \in C, p \in P$$

$$\sum_{i \in C} O_{ip}\, x_{ijp} + \sum_{k \in D} w_{kjp} - \sum_{i \in C} D_{ip}\, x_{jip} - \sum_{k \in D} w_{jkp} = 0 \quad \forall\, j \in D, p \in P$$

$$x_{ijp}, x_{jip} \geq 0 \qquad \forall\, i \in C, j \in D, p \in P$$

$$w_{jkp} \geq 0 \qquad \forall\, j, k \in D, p \in P$$

which is a multicommodity uncapacitated minimum cost network flow problem (Crainic, Dejax and Delorme, 1989).

The complementarity slackness conditions between constraints (P2) and the γ variables are

$$\gamma_{ijp} > 0 \implies y_j = x_{ijp} \quad \text{and} \quad \gamma_{jip} > 0 \implies y_j = x_{jip}$$

and

$$y_j > x_{ijp} \implies \gamma_{ijp} = 0 \quad \text{and} \quad y_j > x_{jip} \implies \gamma_{jip} = 0,$$

for all $i \in C$, $j \in D$ and $p \in P$. Thus, in problem $(P_2(\tilde{\gamma}))$, the $\tilde{\gamma}_{ijp}$ and $\tilde{\gamma}_{jip}$ terms adjust the costs of transportation between customers and depots to account for the fixed, opening and operating, cost of each open depot j. Accordingly, one may interpret these terms as prices to be paid in order to increase the value of y_j, hence to open depot j.

Let $(\tilde{x}, \tilde{w})$ be the optimal solution of problem $(P_2(\tilde{\gamma}))$ and $Z(\tilde{x}, \tilde{w} \mid \tilde{\gamma})$ the corresponding objective function value. Define $\tilde{y}$ as follows:

$$\tilde{y}_j = \underset{i \in C,\, p \in P}{\textit{maximum}} \{ \tilde{x}_{ijp}, \tilde{x}_{jip} \} \quad \forall\, j \in D.$$

Then, $\tilde{y}_j \leq 1$, $\forall\, j \in D$, and $(\tilde{y}, \tilde{x}, \tilde{w})$ is a feasible solution for problem (P). Moreover, a feasible solution for problem (L), $(\bar{y}, \tilde{x}, \tilde{w})$, may be easily obtained by fixing open all depots used in the solution of the multicommodity minimum cost network flow problem: $\bar{y} = \lceil \tilde{y} \rceil$. Consequently, we have

Proposition 3. $Z(\tilde{x}, \tilde{w} \mid \tilde{\gamma}) \leq Z^* \leq Z(\tilde{y}, \tilde{x}, \tilde{w}) \leq Z(\bar{y}, \tilde{x}, \tilde{w})$.

2.2 Dual-Ascent Procedures

The results of the previous subsection may be used to build several dual-ascent procedures to solve problem (P). In particular, problems $(D_1(\tilde{\lambda}))$ and $(P_2(\tilde{\gamma}))$ may be solved alternatively, the results of each problem being assigned as input to the other. A complete iteration of such a procedure consists of the sequential resolution of both problems, the particular procedure being defined by the specific order of resolution of the two problems. Define LB as the current lower bound, BUB the best current upper bound, and

Procedure $FLIP(\tilde{\gamma})$ – Minimum Cost Multicommodity Flow Problem

- Solve problem $(P_2(\tilde{\gamma}))$; let $(\tilde{x}, \tilde{w})$ be the optimal solution, and $(\tilde{u}, \tilde{v}, \tilde{\lambda})$ the corresponding optimal dual variables;
- Compute $\bar{y}_j = \lceil \underset{i \in C,\, p \in P}{maximum} \{ \tilde{x}_{ijp}, \tilde{x}_{jip} \} \rceil \quad \forall\, j \in D$;
- $LB = Z(\tilde{x}, \tilde{w} \mid \tilde{\gamma})$; $BUB = minimum \{ BUB, Z(\bar{y}, \tilde{x}, \tilde{w}) \}$.

and

Procedure $FLOP(\tilde{\lambda})$ – Uncapacitated Multicommodity Location Problem

- Solve problem $(D_1(\tilde{\lambda}))$; let $(\bar{u}, \bar{v}, \bar{\gamma})$ be the optimal solution, and $(\bar{y}, \bar{x})$ the corresponding feasible solution to the multicommodity location problem $(L_1(\tilde{\lambda}))$;
- Solve problem $L(\bar{y}, x, w)$; let $(\bar{x}, \bar{w})$ be the optimal solution;
- $LB = Z(\bar{u}, \bar{v}, \bar{\gamma} \mid \tilde{\lambda})$; $BUB = minimum \{ BUB, Z(\bar{y}, \bar{x}, \bar{w}) \}$.

We may now formally state the two dual-ascent procedure, where ε_1 and ε_2 are suitably small real numbers, and *maxiter* is an integer representing the maximum number of iterations allowed:

Procedure *FLIP-FLOP*

[1] $\tilde{\gamma} = 0$;
$BLB = 0$;
$BUB = \infty$.

[2] $LBLB = BLB$.

[3] Solve the *FLIP* problem.

[4] $\bar{\lambda} = \tilde{\lambda}$ ($u = \tilde{u}$, $v = \tilde{v}$).

[5] Solve the *FLOP* problem.

[6] $\tilde{\gamma} = \bar{\gamma}$.

[7] $BLB = LB$

[8] If $\dfrac{BLB - LBLB}{LBLB} < \varepsilon_1$ or,
$GAP = \dfrac{BUB - BLB}{BLB} < \varepsilon_2$ or,
the number of performed iterations
is equal to *maxiter*,
then STOP; otherwise go to step 2.

Procedure *FLOP-FLIP*

[1] $\bar{\lambda} = 0$;
$BLB = 0$;
$BUB = \infty$.

[2] $LBLB = BLB$.

[3] Solve the *FLOP* problem.

[4] $\bar{\gamma} = \bar{\gamma}$.

[5] Solve the *FLIP* problem.

[6] $\bar{\lambda} = \tilde{\lambda}$ ($u = \tilde{u}$, $v = \tilde{v}$).

[7] $BLB = LB$

[8] If $\dfrac{BLB - LBLB}{LBLB} < \varepsilon_1$ or,
$GAP = \dfrac{BUB - BLB}{BLB} < \varepsilon_2$ or,
the number of performed iterations
is equal to *maxiter*,
then STOP; otherwise go to step 2.

Both procedures require that the dual of a multicommodity simple plant location problem (*FLOP*) be solved to optimality at each iteration. Yet, from a computational point of view, this is not a reasonable alternative. Indeed, in our implementation, we only obtain (see the next section) a "good" feasible solution to problem ($D_1(\bar{\lambda})$), from which a feasible solution to problem ($L_1(\bar{\lambda})$) is extracted (to avoid overcharging the paper, we keep the same notation for these solutions). Moreover, to ensure the monotonicity of the lower bound values, the $\tilde{u}$ and $\tilde{v}$ variables obtained at the end of the minimum cost multicommodity network flow problem (*FLIP*) are used as initial values for the location problem (*FLOP*.) The procedures compute then an increasing series of lower bounds.

Finally, note that the bounds might be further tightened if one observes that, following the resolution of problem ($D_1(\bar{\lambda})$), the values of the γ variables may be changed somewhat

without modifying its (optimal) solution. Indeed, these variables do not appear in the objective function of this problem. Thus, a modification which does not affect its feasibility has no impact on its optimality. But, with higher values for the γ variables, the value of $Z(\bar{x}, \bar{w} \mid \tilde{\gamma})$, corresponding to the optimal objective value of the minimum cost network flow problem $(P_2(\tilde{\gamma}))$, increases; hence, a tighter lower bound computed at the end of the *FLIP* procedure.

The modification of the γ variables thus aims to make the transportation costs between customers and depots (on the existing links) reflect more closely the cost of opening and operating the depots. Consequently, the values of these variables are increased to take out any slack which may exist at the end of the procedure *FLOP* in the dual constraints (D2). Let $e_j, \forall\ j \in D$, be the slack variables in constraints (D2). For the (optimum) solution of problem $(D_1(\bar{\lambda}))$, we then have

$$\sum_{p \in P} \sum_{i \in C} (\bar{\gamma}_{ijp} + \bar{\gamma}_{jip}) + \bar{e}_j = f_j \qquad \forall\ j \in D.$$

Step [6] ([4]) in procedure *FLIP-FLOP (FLOP-FLIP)* then becomes

$$\tilde{\gamma}_{ijp} = \bar{\gamma}_{ijp} + \bar{e}_j \frac{O_{ip}}{\sum_{i \in C,\, p \in P \mid C_{ijp} < \infty} O_{ip} + \sum_{i \in C,\, p \in P \mid C_{jip} < \infty} D_{ip}}$$

$$\tilde{\gamma}_{jip} = \bar{\gamma}_{jip} + \bar{e}_j \frac{D_{ip}}{\sum_{i \in C,\, p \in P \mid C_{ijp} < \infty} O_{ip} + \sum_{i \in C,\, p \in P \mid C_{jip} < \infty} D_{ip}}$$

for all $i \in C$, $j \in D$, $p \in P$, such as C_{ijp} $(C_{jip}) < \infty$.

3. EXPERIMENTATION

The experimentation with the dual-ascent based procedures was conducted on two sets of "random" test problems, and on data based on an actual large scale application.

The aim of the experiments is to determine the behaviour, precision and relative performance of the two dual-ascent heuristics for various types of problems. The main problem characteristics considered are the number of candidate depots and the network dimensions, the number of products, the magnitude of the interdepot traffic generated by the differences between the supply and demand present in each one of the zones of the network, and the relative importance of the fixed depot costs versus the transportation costs. The principal descriptors used to quantify the performance of the two procedures are the speed of convergence, the computation time, and the quality of the solution measured both as the relative gap between the lower and upper bounds, and as the difference between the best upper bound and the optimal solution.

Our main goal was not to obtain the fastest procedure, but rather to determine the most appropriate one among the available alternatives, given the problem characteristics. Consequently, we did not program our own minimum cost network flow algorithm for the FLIP subproblem; instead, we adapted the RNET code (Grigoriadis and Hsu, 1979) to take into account real valued flows and costs (computed in double precision). We have coded, however, a multicommodity version of DUALOC (Erlenkotter, 1978), in order to solve the FLOP subproblem. The "dual-ascent" phase of the algorithm follows Erlenkotter, while the "adjustment" step, as well as the procedure used to obtain a feasible (integer) primal solution, is based on the work of Van Roy and Erlenkotter (1982). Note that we compute only a "good" solution, rather than the optimal one, to the location subproblem. Hence, the remarks of the previous section apply. The procedures are coded in FORTRAN and the experiments were run on a network of SUN Sparc workstations, by using $\varepsilon_1 = \varepsilon_2 = 0.001$

Table 1 displays the typical dimensions of the random-generated test problems that we use. The type of the problem is described by its label: "m" and "l" indicate medium and large-sized problems respectively, N represents the degree of imbalance present in the system (the higher the figure, the larger the required interdepot flows), while P corresponds to the number of products. Note that, for a specific problem, the figures indicated in Table 1 may

Type of Problem	Regular Nodes	Candidate Locations	Regular Arcs (1 Product)	Interdepot Arcs (1 Product)
mz*N* p*P*	125	25	1760	600
lz*N* p*P*	219	44	5260	1892

Table 1. Typical Dimensions for Random Generated Test Problems

vary by a few units. Also, for multicommodity problems, the figures for regular and interdepot arcs are multiplied by the number of products involved. We generate the problems by using the procedure described by Crainic, Delorme and Dejax (1991), who also used some of the same problems to test enumeration algorithms for the location-allocation problem with balancing requirements. It may be useful to recall that the problems are generated by randomly selecting nodes out of a North American road network, and that cost coefficients are based on the (known) distances between these points. Hence, the triangle inequality is ensured for these coefficients.

To complete our experiments, we also used data from an actual application. The original application considers the planning of the land transportation operations of a major international container shipping company operating out of Europe. The problem consists of 12 aggregated types of products, 299 aggregated customer zones, 87 candidate depots, in a geographical zone covering five western European countries. Once the 9 initial transportation modes have been aggregated into the least cost (eventually multimodal) arcs between each pair of nodes, we have a network that contains 4197 interdepot arcs and 32733 arcs between customers and depots. A detailed analysis of this case may be found in Dejax, Crainic and Delorme (1992). In the present paper, we concentrate on the computational performance of the dual-ascent procedures when used to solve this problem. Hence, since the original fixed depot costs are rather low, we also solved problems where the fixed costs were multiplied by 10, 100 and 1000, respectively.

Table 2 displays the results obtained by using the two procedures to solve 42 medium-sized problems. The label 10*x* indicates that we solve again the problem defined on the previous line but with the fixed depot costs multiplied tenfold. For each problem and procedure, the table indicates the iteration where the leveling off of the lower bound occurs, the corresponding computation time (in CPU seconds), the relative gap (in percentage) defined by

the *GAP* formula of the previous section, and the difference (in percentage of the optimal solution value) between the objective value of the best feasible solution found (the best upper bound) and the optimal solution of the problem computed by using a branch-and-bound algorithm (Gendron and Crainic, 1992). The results of the tests performed on large sized problems and on those based on real life data are shown in Table 3.

The first main conclusion of the experimentation is that the two procedures converge extremely fast. We evaluate the speed of convergence of the dual-based procedures by measuring how fast the relative improvement in the value of the lower bound (test performed in step 8 of the procedures) levels off. Indeed, in almost all cases, less than 1% improvement in the lower bound is obtained after the first or the second iteration of the FLIP-FLOP procedure. The FLOP-FLIP procedure generally requires two or three iterations. These results were observed for all our tests, irrespective of the problem size or complication. In practical terms, no further improvement may be expected by performing more iterations. Indeed, we ran the algorithms for ten iterations on all problems and observed either negligible improvements, or no improvement at all.

The two procedures require relatively limited computational effort. We observe that, as expected, computation times increase with the number of products; the effect of the system imbalance factor, as well as that of the problem size, is less marked. One remarks, however, that more "complicated" problems (i.e., more zones and more products) are comparatively easier to solve than simpler ones. This is similar to, but rather less extreme than, the behaviour of the branch-and-bound procedures proposed by Crainic, Delorme and Dejax (1991) for the same type of problems.

The magnitude of the fixed costs has a definite impact both on the computational efficiency of the procedures, and on the quality of the solutions they achieve. On the one hand, the per iteration computation time increases sensibly with the fixed costs, while the number of iterations also goes up, albeit much less markedly. On the other hand, the relative gap tends to increase for higher fixed costs. The fact that the (continuous) minimum cost network flow problem cannot account very precisely for the fixed depot costs explains, at least partially, this behaviour.

Problem	FLIP–FLOP				FLOP–FLIP			
	Iter.	Time	Gap	Diff.	Iter.	Time	Gap	Diff.
mz1p1	1	0.65	0.66	0.00	2	1.48	1.74	0.10
10x	2	3.56	6.93	6.10	2	2.83	5.44	3.87
mz1p2	2	2.99	1.64	0.38	2	1.52	5.64	2.53
10x	2	13.29	6.10	1.90	3	13.58	7.14	1.90
mz1p3	1	1.26	0.67	0.16	3	5.63	1.88	0.32
10x	2	19.40	1.30	0.00	3	22.97	5.20	0.90
mz1p4	1	2.53	1.30	0.00	2	5.63	0.74	0.00
10x	2	12.66	3.01	1.36	3	14.53	7.62	1.71
mz1p5	1	2.73	0.66	0.18	3	12.55	1.47	0.14
10x	2	20.75	1.62	0.00	3	8.54	4.54	0.30
mz1p6	1	4.31	0.30	0.04	2	10.30	0.70	0.01
10x	2	22.80	0.45	0.00	2	24.38	2.15	0.00
mz1p10	1	6.91	0.43	0.05	2	16.23	0.70	0.04
10x	2	100.93	0.98	0.09	3	63.96	3.36	0.34
mz2p1	2	1.54	2.30	1.07	3	2.63	5.78	0.54
10x	4	6.10	11.73	2.82	2	1.94	18.29	2.82
mz2p2	2	3.75	1.51	0.00	3	3.28	3.90	0.48
10x	2	9.80	9.05	3.98	3	9.32	11.58	0.00
mz2p3	2	5.81	1.13	0.50	3	6.95	1.97	0.16
10x	2	13.82	0.78	0.00	3	16.16	6.78	1.02
mz2p4	1	2.24	0.15	0.00	2	6.17	0.66	0.17
10x	2	14.89	2.10	0.94	3	18.40	4.28	0.81
mz2p5	1	2.72	0.97	0.13	3	12.76	1.82	0.16
10x	2	7.61	3.71	0.60	3	7.02	7.86	0.88
mz2p6	1	2.84	0.27	0.06	2	8.34	0.51	0.05
10x	2	39.22	2.07	1.39	3	50.51	2.00	0.19
mz2p10	1	5.15	0.15	0.00	2	18.64	0.61	0.11
10x	2	83.29	1.25	0.69	3	76.68	3.02	0.89
mz4p1	2	1.25	1.85	1.29	3	1.54	6.04	1.54
10x	3	5.35	4.36	1.08	2	2.78	6.40	2.44
mz4p2	2	3.69	0.87	0.03	3	3.23	3.23	0.03
10x	3	7.33	2.01	0.00	3	3.48	6.59	0.00
mz4p3	1	2.20	0.22	0.01	3	7.92	0.70	0.33
10x	2	11.36	2.60	0.91	3	19.23	8.40	2.38
mz4p4	2	6.25	0.65	0.37	3	9.05	1.43	0.54
10x	2	20.05	2.41	0.75	3	10.59	6.79	0.69
mz4p5	1	2.02	0.37	0.23	3	12.39	0.79	0.12
10x	2	20.22	2.25	0.57	3	7.67	6.09	0.51
mz4p6	1	3.22	0.75	0.35	3	10.17	1.14	0.42
10x	2	26.60	1.66	0.00	3	18.02	5.47	0.61
mz4p10	1	5.95	0.19	0.00	2	14.56	0.40	0.00
10x	2	97.43	1.08	0.00	3	70.95	4.64	1.21

Table 2. Results for Random Problems of Medium Size

The two procedures achieve excellent solutions, as illustrated by the relative gaps which are, in general, relatively small. These results are even more significant when compared with those obtained by using a branch-and-bound procedure based on the weak relaxation (determined by aggregating constraints (P2)) of the problem (Crainic, Delorme and Dejax; 1991); after 1000 iterations of the branch-and-bound procedure, the gaps varied from 6.10% (problem mz4p5x10) to 46.90% (problem mz2p1x10). Hence, by using dual-ascent heuristics, the gap may be improved significantly (between 20% and 95% for medium sized problems), even for "difficult" problems, such as mz2p1x10 for which the FLOP-FLIP procedure gives a near optimal solution and a gap some 61% lower than that of the branch-and-bound solution. Finally, one also notes that, in general, the FLIP-FLOP procedure gives more precise solutions than the FLOP-FLIP algorithm.

Problem	γ not adjusted				γ adjusted: $e_j = 0$				γ adjusted: $e_j = f_j$			
	Iter.	Time	Gap	Diff.	Iter.	Time	Gap	Diff.	Iter.	Time	Gap	Diff.
lz1p1	1	4.83	0.57	0.27	1	4.86	0.57	0.27	1	5.53	0.77	0.33
10x	2	44.00	29.45	25.27	2	49.23	24.38	20.45	3	37.47	3.38	0.16
lz1p2	2	22.37	1.12	0.12	2	18.38	0.98	0.12	2	15.67	1.11	0.06
10x	1	22.37	0.90	0.00	1	22.51	0.90	0.00	1	28.22	0.86	0.00
lz3p1	2	6.44	1.34	1.00	2	7.17	1.34	1.00	2	6.76	1.46	1.04
10x	2	21.82	2.70	1.27	2	22.23	2.31	1.27	2	17.76	2.10	0.00
lz3p2	2	20.26	0.96	0.15	2	18.65	0.83	0.13	2	16.69	0.97	0.19
10x	2	62.81	3.15	0.56	2	55.21	2.94	0.38	2	54.32	3.52	0.38
lz5p1	2	6.70	0.77	0.12	2	6.76	1.01	0.44	2	6.71	1.05	0.38
10x	2	12.06	3.62	2.22	2	10.85	3.54	2.22	3	22.82	1.97	0.00
lz5p2	2	16.72	0.68	0.23	2	16.09	0.64	0.19	2	21.72	1.58	1.07
10x	2	22.02	4.94	0.00	3	32.30	3.14	0.00	2	37.10	3.33	0.00
original	1	4.76	0.00	0.00	1	4.82	0.00	0.00	1	46.47	0.10	0.03
10x	1	8.54	0.21	0.03	1	8.78	0.21	0.03	1	38.79	0.48	0.03
100x	2	27.77	1.47	0.16	2	47.71	0.87	0.00	2	60.43	1.17	0.16
1000x	2	47.68	0.95	0.00	2	58.61	0.43	0.00	2	80.81	1.0	0.01

Table 3. FLIP-FLOP with Gamma Adjustments

It is worth noting that the procedures achieve solutions which are optimal or very near optimal. Optimal solutions were obtained by using a branch-and-bound procedure that uses the dual-ascent heuristics to evaluate the lower and upper bounds (Gendron and Crainic, 1992). As the figures displayed in Tables 2 and 3 illustrate, the (feasible) solution that corresponds to the best upper bound is often optimal, or has a very low optimality gap. Note, however, that

the solution which corresponds to the best lower bound is never feasible (integer). Finally, more optimal solutions are obtained by using the FLIP-FLOP procedure, which also generally achieves the smaller optimality gaps. Hence, in general, the FLIP-FLOP procedure is to be preferred to the FLOP-FLIP one: computation times are lower, the solution is obtained more rapidly (in terms of number of iterations), while its quality is superior (in terms of the relative and optimality gaps).

These observations were further corroborated by the results of the second set of experiments. This second experimental phase is aimed at evaluating the impact on the general behaviour and performance of the dual-ascent heuristics of the adjustments to the γ variables (in steps [6] and [4] of procedures FLIP-FLOP and FLOP-FLIP, respectively) to tighten the bounds. Table 3 displays these results for the large size problems, as well as for those derived from the actual application (identified as "original"), for the FLIP-FLOP procedure. We tested two initial adjustments for the "slack" (e_j) variables: at zero or at the fixed cost. The respective results are given in the second and third groups of columns of Table 3.

The conclusions are somewhat mixed. On the one hand, we note that the calculation and utilization of the adjusted γ variables has little influence on the computational effort needed to solve the random problems, while, in general, reducing the relative gap. We also note an increase in the number of problems solved to optimality, especially when the slack variables are initially adjusted at the fixed costs values. On the other hand, the γ adjustment procedures seem couterproductive for the problems derived from the actual application. We plan to further explore this behaviour. In the meantime, the FLIP-FLOP procedure with the adjustment of the γ variables (with initial slacks at fixed costs) seems appropriate for problems with no particular structure. However, the adjustment should be avoided for problems with a structure similar to that of the real-life problems we tested: with several products that differ significantly among themselfs both in the volume and spatial distribution of the demand, and in the density of the specific subnetwork.

It is interesting to note that the FLIP-FLOP dual-ascent procedure has solved to optimality, and in only one iteration, the problem derived from the real application. This emphasizes the behaviour of this type of solution method when applied to the class of location-allocation problems with balancing requirements. The method behaves better when one of its two components, the location or the minimum cost subproblem, "dominates" the other. For example,

in the original problem, fixed costs are very low, hence the flow problem is the most important. On the other hand, multiplying the fixed costs by 1000, makes location the determinant component of the problem, and the optimal solution is obtained. This behaviour is also verified by a simple experiment. We multiplied the costs of the interdepot arcs by 100. Then, the hypothesis mentioned in Section 1 no longer holds, and interdepot balancing movements are no longer interesting. Hence, the problem is now almost equivalent to a location problem and the optimal solution is generally found in one iteration.

Finally, we note that the FLOP-FLIP procedure displays the same general behaviour. Its poorer performance may be explained by the fact that it starts by solving a location problem. Thus, the balancing factor is not taken into account from the beginning. In particular, some depots may be closed even if they might be required to balance the system globally. Hence, the procedure tends towards a less good solution in almost all circumstances.

CONCLUSION

We have presented a dual-ascent based approach for solving multicommodity location-allocation problems with balancing requirements. Two variants have been described and analyzed and both have been shown to solve very efficiently a large gamut of problem types, including problems from actual applications. The FLIP-FLOP procedure, however, consistently outperforms the other and should therefore be preferred. Used as a heuristic, the procedure yields very good solutions and often achieves optimality.

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